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Flux compactifications on Calabi-Yau threefolds

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Abstract: The presence of RR and NS three-form fluxes in type IIB string compactification on a Calabi-Yau orientifold gives rise to a nontrivial superpotential W for the dilaton and complex structure moduli. This superpotential is computable in terms of the period integrals of the Calabi-Yau manifold. In this paper, we present explicit examples of both supersymmetric and nonsupersymmetric solutions to the resulting 4d $\mathcal{N} = 1$ supersymmetric no-scale supergravity, including some nonsupersymmetric solutions with relatively small values of W. Our examples arise on orientifolds of the hypersurfaces in $WP^4_{1,1,1,1,4}$ and $WP^4_{1,1,2,2,6}$. They serve as explicit illustrations of several of the ingredients which have played a role in the recent proposals for constructing de Sitter vacua of string theory.

Keywords: F-Theory, Superstring Vacua.

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1. Introduction

Given the vast array of possible string compactifications to 4d, it is very useful to find large classes of constructions which can be studied systematically. One of the most interesting questions regards the detailed structure of the potential for the plethora of moduli fields that typically arise. In the most familiar case of Calabi-Yau compactifications, these moduli include the complex structure and Kähler moduli of the Calabi-Yau space, and the dilaton or string coupling constant. Knowledge of the potential for these moduli is crucial in making concrete models of particle physics, in designing cosmological scenarios in string theory, and in understanding what if anything string theory says about the cosmological constant problem.

It has been realized over the past several years that in fact in generic compactifications of string theory to four dimensions, one is allowed to turn on fluxes of some of the p-form RR and NS fields in the compact dimensions (see e.g. [1]–[8]; dual descriptions of some very simple flux compactifications appear in [9]). We shall focus on the specific case of the type IIB theory on a Calabi-Yau orientifold, which preserves 4d $\mathcal{N} = 1$ supersymmetry at the KK scale. The relevant background fluxes are those of the RR three-form field strength $F_{(3)}$ and the NS three-form field strength $H_{(3)}$. Given a choice of these fluxes, i.e. of two integral three-forms obeying a tadpole condition determined by the precise orientifold, one can compute the superpotential W very explicitly in terms of periods of the holomorphic three-form Ω [2, 10]. An appropriate framework for analyzing these solutions in some detail was developed in [5], and explicit examples involving tori and K3 surfaces were studied in detail in [6]–[8]. The possibility of constructing models with significant warping was described in [11, 5] (the construction of [5] follows closely the work of [12]; see also [3] for earlier papers about constructing warped flux compactifications). Recently, models with chiral low-energy gauge theories were discussed in roughly this framework [13, 14], and a catalogue of flux-induced soft susy breaking terms on D3-branes was derived [15]. An up to date review of this subject can be found in [16], and work developing the relevant gauged supergravities to describe this class of compactifications can be found in [17].

Given the rather explicit form of the Gukov-Vafa-Witten superpotential that controls much of the dynamics of the moduli in these compactifications, it is reasonable to hope that one can understand the properties of the solutions (at least in the leading-order no-scale supergravity approximation) rather explicitly. However, to date, the only (compact) examples presented in complete detail have involved toroidal orientifolds or Calabi-Yau spaces with reduced holonomy. Here, we present some explicit solutions of the IIB flux equations for orientifolds of "generic" Calabi-Yau threefolds, whose holonomy fills out $SU(3)$. This is of more than academic interest: such examples are closely related to some proposals for constructing de Sitter vacua in string theory [18, 19], and for more precisely estimating the number of metastable string vacua [20].

We will find two surprises in our analysis. First, we will find that supersymmetric solutions of the flux equations do exist. Given an elementary counting argument which we will review below, this is by itself somewhat surprising. Perhaps more importantly, we will find that simple nonsupersymmetric solutions to the flux equations (still at vanishing potential $V = 0$ in the no-scale approximation, as described in [5]) with small values of W also exist. This is a bit surprising given the small numbers of fluxes we will be turning on. These examples provide support for the assertion in e.g. [18] that by discretely tuning the choice of fluxes in manifolds with large b_3, one can attain small values of W.[1]

The organization of this paper is as follows. In section 2, we describe the basic facts about the two models (which we call model A and model B) that we will be studying — the threefold geometries, the relevant orientifold actions, and the lift to an F-theory description. We also describe the special (small) subclass of fluxes we will be turning on,

[1] Of course this is a meaningful notion only after one has fixed the Kähler invariance, otherwise one should specify something physical like the gravitino mass. We will describe the conventions in which we desire "small W" below, they coincide with those in [18].

and the symmetries of the resulting potential which guarantee that we can consistently solve the equations with many of the CY moduli frozen at a special symmetric locus. This saves us from having to solve the Picard-Fuchs equations for hundreds of independent periods in the two models. In section 3, we give a more precise formulation of the problems of interest, and we present details about the period integrals in the two models. In section 4, we give examples of supersymmetric solutions in model B. In section 5, we give examples of nonsupersymmetric solutions in both models, including some with small W. We close with a discussion in section 6. In two appendices A and B, we include more details about various computations in the two models.

2. The two models of interest

2.1 The Calabi-Yau threefolds

We will be studying orientifolds of two different Calabi-Yau threefolds. Model A will be constructed starting with the threefold M_A which arises as a hypersurface in $WP^4_{1,1,1,1,4}$

$$4x_0^2 + x_1^8 + x_2^8 + x_3^8 + x_4^8 - 8\psi x_0 x_1 x_2 x_3 x_4 = 0\,. \tag{2.1}$$

This threefold has $h^{1,1} = 1$ and $h^{2,1} = 149$. It served as one of the first examples of mirror symmetry [21]–[23], generalizing the seminal work of [24] on the quintic. After taking the quotient by the maximal group of scaling symmetries as in the Greene-Plesser construction of mirror manifolds [25], the modulus ψ describes the single complex structure modulus of a mirror manifold W_A. Then, the classical geometry of complex structure deformations of W_A reproduces the quantum Kähler moduli space of M_A. However we will be interested not in the mirror, but in M_A itself. In this context, there are many other terms that could appear deforming the complex structure in (2.1); we explain why it will be consistent to neglect these deformations in section 2.3. We will also describe the production of an appropriate orientifold in the next subsection.

Our second model, model B, is based on a Calabi-Yau threefold with $h^{1,1}(M_B) = 2$ and $h^{2,1}(M_B) = 128$. The manifold arises as a (resolution of) a hypersurface in $WP^4_{1,1,2,2,6}$

$$x_0^2 + x_1^{12} + x_2^{12} + x_3^6 + x_4^6 - 12\psi x_0 x_1 x_2 x_3 x_4 - 2\phi x_1^6 x_2^6 = 0\,. \tag{2.2}$$

It was also studied as one of the first examples of mirror symmetry in two-parameter models (its mirror W_B has a two-parameter complex structure moduli space), in [26, 27]. In addition it played a role as one of the first examples of $\mathcal{N} = 2$ heterotic/type II string duality [28, 29]. The restriction to the two moduli ϕ and ψ is natural in the Greene-Plesser construction of mirror symmetry, where they parametrize the subspace of the moduli space of M_B which is invariant under the maximal group of scaling symmetries. Again, since we will be interested in M_B and not in its mirror, we could in principle add many additional terms to (2.2); we shall explain their absence below in section 2.3.

2.2 The orientifolds

We are interested in $\mathcal{N} = 1$ compactifications of the type IIB theory on M_A and M_B. To break the symmetry from $\mathcal{N} = 2$ to $\mathcal{N} = 1$, we must orientifold. The orientifolds we study will fall in the class described in [5], and can in fact be produced by Sen's construction [30]

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which relates Calabi-Yau fourfold compactifications of F-theory to IIB orientifolds. In fact, the fastest way for us to compute the relevant properties of the orientifolds will be to follow Sen's procedure, and specify the F-theory fourfolds.

For model A, consider the fourfold X_A given by the Calabi-Yau hypersurface in $WP^5_{1,1,1,1,8,12}$. This is model 5 in [31, table B.4]. Following the procedure of [30], one immediately sees that it reduces to an orientifold of M_A in an appropriate limit. It has $\chi(X_A) = 23,328$, which means that in the IIB picture there will be a tadpole condition [32]

$$N_{D3} + N_{\rm flux} = \frac{\chi(X_A)}{24} = 972\,. \tag{2.3}$$

Here, N_{D3} is the number of space-filling D3 branes one chooses to insert, and $N_{\rm flux}$ is the D3 brane charge carried by the $H_{(3)}$ and $F_{(3)}$ fluxes.

For model B, the fourfold X_B is given by the Calabi-Yau hypersurface in $WP^5_{1,1,2,2,12,18}$, model 21 in [31, table B.4]. Since $\chi(X_B) = 19728$, there will be 822 units of D3 brane charge to play with in this model. Again following [30], one sees that in an appropriate limit, it becomes an orientifold of type IIB on M_B.

What the observations of this subsection teach us is that appropriate $\mathcal{N} = 1$ orientifolds of M_A and M_B do exist, with specified (rather large!) amounts of D3 brane charge that must be inserted (via fluxes or space-filling branes) to satisfy the tadpole condition. In fact, the Sen construction is consistent with producing orientifolds on the loci of complex structure moduli space specified in (2.1) and (2.2). To see this, one simply observes that the Sen orientifold action amounts to taking $x_0 \to -x_0$, composed with worldsheet orientation reversal. It may be confusing that the monomial $x_0x_1x_2x_3x_4$ appears in (2.1) and (2.2) (since it is not invariant). However using the ring relations (setting the partial derivatives of the defining equation to zero), this can be re-expressed in terms of $(x_1x_2x_3x_4)^2$ with an extra factor of ψ, and the deformation is manifestly invariant. More explicitly, in model A for example, one can define a new coordinate $\tilde{x}_0 = x_0 - 4\psi x_1x_2x_3x_4$ and express the defining equation (2.1) in terms of this variable. Then only ψ^2 will appear in the defining polynomial, and the orientifold action will identify $\tilde{x}_0 \to -\tilde{x}_0$. In the presentation given, ψ appears, while in the manifestly invariant prescription only ψ^2 appears. There are identifications on the ψ moduli space that mean that $\psi \to -\psi$ is a symmetry in both models (we will say more about this when we discuss the periods), and in the presentation of the manifolds in (2.1) and (2.2), one should take Sen's prescription to also act with the modular symmetry $\psi \to -\psi$. The reader who finds this confusing is advised to think instead in the picture with the deformation parametrized by $\psi^2(x_1x_2x_3x_4)^2$.

One last comment. The three-form fluxes we turn on are allowed by the orientifold symmetry. The NS and RR two forms, B_2, C_2, have odd intrinsic parity under the orientifold symmetry, and the relevant three-cycles are also odd.[2]. Thus turning on the three forms H_3, F_3, along these three cycles is allowed.

[2]One quick way to see this is as follows. In the definitions of [22, 26, 33], Ω, the holomorphic three form, is even under the orientifold symmetry. From, eq. (3.10), (3.19), we see that the periods are odd, since $\psi \to -\psi$. This shows that the three-cycles are odd.

2.3 Special loci and symmetries

In both model A and model B, there are many complex structure deformations (even including only those which are preserved by the orientifold action). Turning on arbitrary fluxes, the calculation of the flux superpotential would then require a solution of the Picard-Fuchs equations for a vast number of periods. We are not going to proceed in this manner.

Instead, we make the following simple observation. The special families of defining equations we have written down in (2.1) and (2.2) are invariant under large groups of global symmetries. The symmetry group is $\mathcal{G}_A = \mathbb{Z}_2 \times \mathbb{Z}_8^2$ for model A and $\mathcal{G}_B = \mathbb{Z}_2 \times \mathbb{Z}_6^2$ for model B. All deformations which we have not included explicitly in (2.1) and (2.2) transform nontrivially under $\mathcal{G}_{A,B}$. As argued in e.g. [34, section 3] (where the example of the quintic is discussed in detail), this means that the Picard-Fuchs equations simplify greatly, if one is interested only in a subset of the periods. Namely, in model A, there are four periods which will coincide with those of W_A, and in which the other moduli of M_A (which do not appear in (2.1)) can only appear with high enough powers to maintain $\mathcal{G}_A$ invariance. Similarly, in model B, there are six periods which will coincide with those of W_B, and in which the other moduli of M_B will only appear with high enough powers to maintain $\mathcal{G}_B$ invariance.

Roughly speaking, what happens in model A is that there is a four-dimensional subspace of $H_3(M_A)$ which is the homology dual to the four dimensional subspace in $H^3(M_A)$ spanned by the (2,1) form associated to ψ, it's dual (1,2) form, and the (3,0) and (0,3) forms. One can compute the periods in this subspace of $H_3(M_A)$ and they depend only on higher powers of the deformations absent in (2.1), due to $\mathcal{G}_A$ invariance. Similar remarks apply to model B, where however there are six relevant periods instead of four (since one has two deformations, ϕ and ψ).

The physical interpretation of this is clear. Suppose we turn on only fluxes consistent with the $\mathcal{G}_{A,B}$ symmetries. Then certain terms (low order terms in the $\mathcal{G}$-charged moduli) are forbidden from appearing in the flux superpotential. The moduli which appear only at higher order in W can be consistently set to zero (as we have done in the defining equations (2.1) and (2.2)) because of the symmetry. They will generically be constrained by a higher order potential, which is guaranteed to vanish at their origin. Since this only holds if we turn on a restricted set of fluxes which maintain the $\mathcal{G}$ invariance, we can choose fluxes only through four three-cycles in model A and six in model B. These are simply related to the cycles which appear in computing the periods of the mirror manifolds W_A and W_B.[3]

For these reasons, in further discussion of the models, we shall always set the complex structure moduli except ψ in model A and (ϕ, ψ) in model B to be at their origin (where the $\mathcal{G}$ symmetries are unbroken). We shall also neglect the dependence of the periods on these moduli, since as we have explained, it is of high enough order that the equations for these other moduli cannot obstruct solutions on the symmetric locus.

[3]One difference one must be careful to account for involves factors of $|\mathcal{G}|$ in the proper normalization of the periods over integral cycles.

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3. Detailed structure of the models

3.1 Basic facts common to both models

Homology and cohomology bases. We will work with a symplectic homology basis for the subspaces of H_3 of interest to us. The basis of three-cycles A_a and B^a ($a = 1, 2$ for model A and $a = 1, 2, 3$ for model B) and the basis for integral cohomology α_a and β^a satisfy

$$\int_{A_a} \alpha_b = \delta^a_b\,, \qquad \int_{B^b} \beta^a = -\delta^a_b\,, \qquad \int_M \alpha_a \wedge \beta^b = \delta^b_a\,. \tag{3.1}$$

The holomorphic three form can be represented in terms of periods in this basis as follows:

$$\int_{A_a} \Omega = z^a\,, \qquad \int_{B^a} \Omega = \mathcal{G}_a\,, \qquad \Omega = z^a \alpha_a - \mathcal{G}_a \beta^a\,. \tag{3.2}$$

In addition

$$\int_M \Omega \wedge \bar{\Omega} = \bar{z}^a \mathcal{G}_a - z^a \bar{\mathcal{G}}_a = \bar{z}^a \frac{\partial \mathcal{G}}{\partial z^a} - z^a \frac{\partial \overline{\mathcal{G}}}{\partial \bar{z}^a} = -\Pi^\dagger \cdot \Sigma \cdot \Pi\,. \tag{3.3}$$

Here, we have introduced the prepotential $\mathcal{G}(z^1, z^2)$, the period vector Π (whose entries are the periods (3.2)), and the matrix

$$\Sigma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \tag{3.4}$$

whose entries are two by two matrices. This structure is common to all Calabi-Yau compactifications, see e.g. [35].

Fluxes, superpotential and Kähler potential. The NS and RR fluxes admit the following quantization condition

$$F_{(3)} = (2\pi)^2 \alpha' (f_a[B_a] + f_{a+k}[A_a])\,, \qquad H_{(3)} = (2\pi)^2 \alpha' (h_a[B_a] + h_{a+k}[A_a]) \tag{3.5}$$

with integer f_i and h_i. Here $k = 2$ for model A and $k = 3$ for model B, and a runs over $1, 2$ for model A and $1, 2, 3$ for model B. Here we also used the notation $[A_a] = \alpha_a$ and $[B_a] = \beta_a$. Using this notation, we find the following expression for N_{flux}

$$N_{\text{flux}} = \frac{1}{(2\pi)^4 (\alpha')^2} \int_M H_{(3)} \wedge F_{(3)} = f^T \cdot \Sigma \cdot h\,. \tag{3.6}$$

The superpotential is given by

$$W = \int_M (F_{(3)} - \tau H_{(3)}) \wedge \Omega = (2\pi)^2 \alpha' (f \cdot \Pi - \tau h \cdot \Pi)\,. \tag{3.7}$$

The Kähler potential for the dilaton-axion and complex moduli is given by

$$K = K_\tau + K_{\text{c.m.}} = -\ln(-i(\tau - \bar{\tau})) - \ln\left(i \int_M \Omega \wedge \bar{\Omega}\right) = -\ln(-i(\tau - \bar{\tau})) - \ln(-i\Pi^\dagger \cdot \Sigma \cdot \Pi)\,, \tag{3.8}$$

where c.m. is ψ for model A and (ψ, ϕ) for model B. For more discussion of the low-energy effective action of these IIB orientifolds, see e.g. [5].

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Conditions for solutions. The supersymmetry conditions for flux vacua are given by

$$W = 0\,, \qquad D_\tau W = 0\,, \qquad e^K G^{a\bar{b}} D_a W \overline{D_b W} = 0\,, \tag{3.9}$$

where for model A $a, b = \psi$, and for model B they run over ϕ, ψ. We have kept the e^K in (3.9) because given the conventions for normalizing Ω in e.g [22, 26], this factor can sometimes make a difference.

If one wishes to find no-scale vacua without supersymmetry, the conditions (3.9) are relaxed; one need not impose $W = 0$. In such solutions the potential still vanishes at tree-level, but there are non-vanishing F-auxiliary VEVs for some Kähler moduli [5].

3.2 Periods for model A

In [22] (following [24]), the relevant periods for model A are given as follows. In the Picard-Fuchs basis, the fundamental period is given by

$$w_0(\psi) = (2\pi i)^3 \frac{\pi}{8} \sum_{n=1}^{\infty} \frac{1}{\Gamma(n) \prod_{i=0}^{4} \Gamma(1 - \frac{n}{8}\nu_i)} \frac{\exp(i\frac{\pi}{8}7n)}{\sin(\frac{\pi n}{8})} (4\psi)^n\,. \tag{3.10}$$

This expression is valid for $|\psi| < 1$. Here the ν_i are the weights of the WP^4, i.e. $1, 1, 1, 1, 4$. We will choose another gauge and normalization in comparison with [24, 22]. The gauge of [24, 22] is convenient for considering the fundamental period in the vicinity of $\psi = \infty$. For the case $\psi = 0$ at hand, it is useful to make a gauge transformation of the holomorphic three form and corresponding transformation of the Kähler potential

$$\Omega(\psi) \to \frac{1}{\psi}\,\Omega(\psi)\,, \qquad K_\psi \to K_\psi + \ln|\psi|^2\,. \tag{3.11}$$

Also, since we are interested in the orientifold of M_A, not in the mirror, the normalization of the fundamental period differs by $|\mathcal{G}_A|$ in comparison with [24, 22]. This just follows from the definition of the fundamental period, which is given by an integral on the cycle $|x_i| = \delta$ for $i = 0, \ldots, 4$ and small δ.

In terms of the fundamental period, a basis for the periods is given by

$$w_A^T = \frac{1}{\psi}(w_0(\alpha^2\psi), w_0(\alpha\psi), w_0(\psi), w_0(\alpha^7\psi))\,, \tag{3.12}$$

where $\alpha = \exp(\pi i/4)$.

Now, we are really interested in the periods in a symplectic basis. The periods in the symplectic basis $\Pi_A^T = (\mathcal{G}_1, \mathcal{G}_2, z^1, z^2)$ can be expressed in terms of those in the Picard-Fuchs basis by means of a linear transformation:

$$\Pi_A = m_A \cdot w_A\,, \tag{3.13}$$

where the matrix m_A is given by

$$m_A = \begin{pmatrix} -1/2 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 0 \end{pmatrix}. \tag{3.14}$$

It follows from these formulae that in the vicinity of $\psi = 0$, we can expand the period vector as

$$\Pi_A = c_0 p_0 + c_2 p_2 \psi^2 + c_4 p_4 \psi^4 + \cdots . \tag{3.15}$$

Here the vectors p_k are given by

$$p_k = \; m_A \cdot \tilde{p}_k \tag{3.16}$$

with

$$\tilde{p}_k^T = (\alpha^{2(k+1)}, \alpha^{(k+1)}, 1, \alpha^{7(k+1)}) , \tag{3.17}$$

and the constants c_k are as follows

$$\begin{aligned} c_0 &= (2\pi i)^3 \frac{\sqrt{\pi}}{2\Gamma^4(7/8)} \frac{\exp(7\pi i/8)}{\sin(\pi/8)} , \\ c_2 &= -(2\pi i)^3 \frac{2\sqrt{\pi}}{\Gamma^4(5/8)} \frac{\exp(5\pi i/8)}{\sin(3\pi/8)} , \\ c_4 &= (2\pi i)^3 \frac{4\sqrt{\pi}}{\Gamma^4(3/8)} \frac{\exp(3\pi i/8)}{\sin(5\pi/8)} . \end{aligned} \tag{3.18}$$

We kept the fourth order terms here in part to show that they are small compared to the zeroth and second order terms as long as $|\psi| \ll 1$, but they will also play an important role in our nonsupersymmetric solutions in section 5. The solutions we will present there, will be valid for small $|\psi|$ (where, in our examples, the ψ modulus will be stabilized in a self-consistent approximation).

3.3 Periods for model B

In [33, 26] the following power series expansions for the periods of this threefold are given. Define a fundamental period

$$w_0(\psi, \phi) = (2\pi i)^3 \frac{1}{6} \sum_{n=1}^{\infty} \frac{(-1)^n \Gamma(n/6)}{\Gamma(n)\Gamma^2(1-n/6)\Gamma(1-n/2)} (12\psi)^n u_{-n/6}(\phi) , \qquad \left| \frac{864\psi^6}{\phi \pm 1} \right| < 1 . \tag{3.19}$$

Here the u_vs are functions of ϕ which are given in e.g. [26]. As with model A, we perform a gauge transformation rescaling the holomorphic three-form by $1/\psi$, and redefine the fundamental period by multiplying it by $|\mathcal{G}_B|$. Then in [33, 26] they find a six dimensional basis for the periods, given by

$$w_B^T = \frac{1}{\psi} \left(w_0(\psi, \phi), w_0(\alpha\psi, -\phi), w_0(\alpha^2\psi, \phi), w_0(\alpha^3\psi, -\phi), w_0(\alpha^4\psi, \phi), w_0(\alpha^5\psi, -\phi) \right) , \tag{3.20}$$

where $\alpha = \exp(\pi i/6)$.

The periods in a symplectic basis $\Pi_B^T = (\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, z^1, z^2, z^3)$ can be expressed in terms of the Picard-Fuchs basis of periods w by a linear transformation $\Pi_B = m_B \cdot w_B$, where

$$m_B = \begin{pmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 3/2 & 3/2 & 1/2 & 1/2 & -1/2 & -1/2 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1/2 & 0 & 1/2 & 0 & 1/2 & 0 \\ 1/2 & 1/2 & -1/2 & 1/2 & -1/2 & 1/2 \end{pmatrix}. \tag{3.21}$$

In the vicinity of $\psi = 0$ and some regular locus for ϕ, we can expand the periods (3.19) as follows

$$\Pi_B = c_0 p_0 + c_2 p_2 \psi^2 + \cdots\,, \tag{3.22}$$

where c_0 and c_2 are given by

$$c_0 = -(2\pi i)^3 \frac{4\sqrt{\pi}}{\Gamma^3(5/6)}\,, \qquad c_2 = (2\pi i)^3 (12)^2 \frac{1}{2\pi}\,, \tag{3.23}$$

and the vectors p_0 and p_2 are given by

$$\begin{aligned} p_0 &= m_B \cdot (u_{-1/6}(\phi)\tilde{p}_{01} + \alpha u_{-1/6}(-\phi)\tilde{p}_{02}) \equiv u_{-1/6}(\phi) p_{01} + \alpha u_{-1/6}(-\phi) p_{02}\,, \\ p_2 &= m_B \cdot (u_{-1/2}(\phi)\tilde{p}_{21} + i u_{-1/2}(-\phi)\tilde{p}_{22}) \equiv u_{-1/2}(\phi) p_{21} + i u_{-1/2}(-\phi) p_{22}\,, \end{aligned} \tag{3.24}$$

where

$$\begin{aligned} \tilde{p}_{01}^T &= (1, 0, \alpha^2, 0, \alpha^4, 0)\,, \qquad & \tilde{p}_{02}^T &= (0, 1, 0, \alpha^2, 0, \alpha^4)\,, \\ \tilde{p}_{21}^T &= (1, 0, -1, 0, 1, 0)\,, \qquad & \tilde{p}_{22}^T &= (0, 1, 0, -1, 0, 1)\,. \end{aligned} \tag{3.25}$$

This completes our discussion of the periods for the two models. We will refer back to the formulae from these sections as the need arises, when specifying our solutions.

4. Supersymmetric solutions in model B

In this section we will study supersymmetric solutions in model B.

Supersymmetry conditions for model B. The Kähler potential and metric have the following behavior for small ψ at regular points in the ϕ moduli space

$$K(\psi, \phi) \sim 1\,, \qquad G_{\psi\bar{\psi}} \sim \psi\overline{\psi}\,, \qquad G_{\psi\bar{\phi}} \sim \psi\overline{\psi}^2\,, \qquad G_{\phi\bar{\psi}} \sim \overline{\psi}\psi^2\,, \qquad G_{\phi\bar{\phi}} \sim 1\,. \tag{4.1}$$

This means that terms with mixed ψ and ϕ derivatives do not appear in the potential. Henceforth the supersymmetry conditions for flux vacua (3.9) are

$$W = 0\,, \qquad \partial_\tau W = 0\,, \qquad \frac{1}{\psi}\partial_\psi W = 0\,, \qquad \partial_\phi W = 0\,. \tag{4.2}$$

This gives the following supersymmetry conditions

$$f \cdot p_{01} = 0\,, \qquad f \cdot p_{02} = 0\,, \qquad h \cdot p_{01} = 0\,, \qquad h \cdot p_{02} = 0\,, \qquad (f - \tau h) \cdot p_2 = 0\,. \tag{4.3}$$

Explicit solutions. The first four conditions of (4.3) are satisfied for rank two degenerate families of vectors

$$f = \begin{pmatrix} -2f_2 \\ f_2 \\ f_3 \\ -4f_2 - 2f_3 \\ -f_3 \\ 0 \end{pmatrix}, \qquad h = \begin{pmatrix} -2h_2 \\ h_2 \\ h_3 \\ -4h_2 - 2h_3 \\ -h_3 \\ 0 \end{pmatrix}. \tag{4.4}$$

For these families N_{flux} (3.6) is given by

$$N_{\text{flux}} = 3(f_2 h_3 - h_2 f_3)\,. \tag{4.5}$$

The last condition of (4.3) fixes the dilaton-axion to the following value

$$\tau(\phi) = \frac{u_{-\frac{1}{2}}(\phi)(f_2 + f_3) + i u_{-1/2}(-\phi) f_2}{u_{-1/2}(\phi)(h_2 + h_3) + i u_{-1/2}(-\phi) h_2}\,, \tag{4.6}$$

where the $u_{-\frac{1}{2}}(\phi)$ function (given in [26]) is

$$u_{-\frac{1}{2}}(\phi) = \frac{1}{\sqrt{2}\pi} \int_{-1}^{1} d\zeta \frac{1}{\sqrt{(1-\zeta^2)(\phi - \zeta)}}\,. \tag{4.7}$$

In this model we have a moduli space of supersymmetric vacua parametrized by ϕ, with singularities at complex codimension one (for instance on the locus $\phi^2 = 1$).

5. Nonsupersymmetric solutions

5.1 Model A

We obtain solutions to the classical supergravity equations for model A in this section. These solutions break supersymmetry, but the scale of supersymmetry breaking is somewhat small compared to the string scale.

The essential idea behind finding these solutions is the following. We will work in the vicinity of the $\psi = 0$ point in moduli space, eq. (3.10). It will turn out that obtaining a supersymmetric solution at $\psi = 0$ requires that the ratio of two fluxes is an irrational number. This condition cannot be met since the fluxes are quantized to take integer values. However, it is well known that an irrational number can be arbitrarily well approximated by a rational p/q. So by discretely tuning the fluxes we will obtain approximately supersymmetric solutions in the vicinity of $\psi = 0$.

We expect a similar strategy will be more widely useful in the vicinity of other points in moduli space and also for other Calabi-Yau compactifications. In the present example, given the restriction on the total flux which can be turned on, (3.6), the flux integers p, q cannot be taken to be very big, and one can do only moderately well in lowering the susy breaking scale. In other cases where the total value of flux can be larger, one would expect that the flux integers can be made bigger and the approximation to the irrational number can be quite good, resulting in a small scale of supersymmetry breaking. Perhaps more importantly, for simplicity we have turned on fluxes along only four three-cycles in this analysis. When more fluxes are turned on one would expect to do better in terms of lowering the supersymmetry breaking scale.

The analysis below proceeds in three steps. We first examine the requirements for a supersymmetric solution at $\psi = 0$. We then consider the supersymmetry conditions up to $\mathcal{O}(\psi^2)$ and show that for appropriately chosen fluxes they can be met. Finally, we consider the analysis to higher orders in ψ and show that the solution breaks supersymmetry at order $\mathcal{O}(\psi^4)$.

1. Conditions for SUSY Solution at $\psi = 0$.

   As discussed previously, the fluxes can be expanded in an integral cohomology basis

$$F_{(3)} = (2\pi)^2\alpha'(f_a[B_a] + f_{a+2}[A_a])\,, \qquad H_{(3)} = (2\pi)^2\alpha'(h_a[B_a] + h_{a+2}[A_a])\,. \tag{5.1}$$

   The superpotential then becomes

$$W_A = f \cdot \Pi_A - \tau h \cdot \Pi_A\,. \tag{5.2}$$

   (We are neglecting a factor of $(2\pi)^2\alpha'$ in the normalisation for W_A for now, (3.7), this will be restored towards the end of the section when we calculate the scale of supersymmetry breaking). The susy conditions provided by (3.9) are

$$f \cdot \Pi_A = 0\,, \qquad h \cdot \Pi_A = 0\,, \tag{5.3}$$

   and

$$\frac{1}{\psi}(f - \tau h) \cdot \partial_\psi \Pi_A = 0\,. \tag{5.4}$$

   The last equation, (5.4), should be understood as a limiting value at $\psi = 0$. As we will see later on in this section, in the vicinity of $\psi = 0$, the metric $G_{\psi\bar{\psi}} \sim \psi\bar{\psi}$. Eq. (5.4) then follows from (3.9).

   Keeping terms up to $\mathcal{O}(\psi^2)$ we find

$$\begin{aligned} c_0\tilde{h} \cdot \tilde{p}_0 + c_2\psi^2\tilde{h} \cdot \tilde{p}_2 &= 0\,, \\ c_0\tilde{f} \cdot \tilde{p}_0 + c_2\psi^2\tilde{f} \cdot \tilde{p}_2 &= 0\,, \\ \tilde{f} \cdot \tilde{p}_2 - \tau\tilde{h} \cdot \tilde{p}_2 &= 0\,. \end{aligned} \tag{5.5}$$

   Here for convenience, we redefined the vectors so that

$$\tilde{f} = f \cdot m_A\,, \qquad \tilde{h} = h \cdot m_A\,, \tag{5.6}$$

   with the integral flux vectors given by

$$f = (f_1, f_2, f_3, f_4)\,, \tag{5.7}$$

$$h = (h_1, h_2, h_3, h_4)\,. \tag{5.8}$$

For use below we note that $\tilde{f}$ and $\tilde{h}$ are given in terms of f and h as

$$\tilde{f}^T = \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \\ \tilde{f}_3 \\ \tilde{f}_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -f_1 - 2f_3 \\ -f_1 + 2f_4 \\ f_1 - 2f_2 + 6f_3 - 2f_4 cr f_1 + 4f_3 \end{pmatrix}, \tag{5.9}$$

$$\tilde{h}^T = \begin{pmatrix} \tilde{h}_1 \\ \tilde{h}_2 \\ \tilde{h}_3 \\ \tilde{h}_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -h_1 - 2h_3 \\ -h_1 + 2h_4 \\ h_1 - 2h_2 + 6h_3 - 2h_4 \\ h_1 + 4h_3 \end{pmatrix}. \tag{5.10}$$

Also, the total contribution to the D3 charge tadpole from the fluxes is given by

$$\begin{aligned} N_{\text{flux}} &= f \cdot \Sigma \cdot h^T \\ &= 2(\tilde{f}_4\tilde{h}_2 - \tilde{f}_2\tilde{h}_4 + \tilde{f}_3\tilde{h}_1 - \tilde{f}_1\tilde{h}_3) + \tilde{f}_1(\tilde{h}_2 + \tilde{h}_4) + \tilde{f}_2(\tilde{h}_3 - \tilde{h}_1) + \\ &\quad + \tilde{f}_3(\tilde{h}_4 - \tilde{h}_2) - \tilde{f}_4(\tilde{h}_1 + \tilde{h}_3)\,. \end{aligned} \tag{5.11}$$

Since f_i and h_i must be integer we note that $\tilde{f}_i$'s and $\tilde{h}_i$'s are rational numbers in general.

Now we are ready to consider the requirements that need to be met for a susy solution at $\psi = 0$. This imposes two conditions on the flux

$$\tilde{f} \cdot \tilde{p}_0 = 0\,, \tag{5.12}$$

and

$$\tilde{h} \cdot \tilde{p}_0 = 0\,. \tag{5.13}$$

The dilaton-axion is then given by

$$\tau = \frac{\tilde{f} \cdot \tilde{p}_2}{\tilde{h} \cdot \tilde{p}_2}\,. \tag{5.14}$$

Using (3.17), (5.12) takes the form

$$i\tilde{f}_1 + \alpha\tilde{f}_2 + \tilde{f}_3 - i\alpha\tilde{f}_4 = 0\,. \tag{5.15}$$

To simplify the analysis we will consider from here on fluxes which meet the condition

$$\tilde{f}_4 = 0\,, \qquad \tilde{f}_3 = \tilde{f}_1\,. \tag{5.16}$$

Now (5.12) becomes

$$\frac{\tilde{f}_1}{\tilde{f}_2} = -\frac{1}{\sqrt{2}}\,. \tag{5.17}$$

As noted above $\tilde{f}_i$ must be rational, so (5.17) cannot be met.

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Similarly, (5.13) takes the form

$$i\tilde{h}_1 + \alpha\tilde{h}_2 + \tilde{h}_3 - i\alpha\tilde{h}_4 = 0\,. \tag{5.18}$$

Again for easy of analysis we consider the case where

$$\tilde{h}_2 = 0\,, \qquad \tilde{h}_3 = -\tilde{h}_1\,. \tag{5.19}$$

Then (5.13) becomes

$$\frac{\tilde{h}_1}{\tilde{h}_4} = \frac{1}{\sqrt{2}}\,. \tag{5.20}$$

This condition again can not be met.

Thus, we cannot have a supersymmetric solution at $\psi = 0$ in this model.

2. A SUSY solution to $\mathcal{O}(\psi^2)$.

   We now show that to $\mathcal{O}(\psi^2)$ the SUSY conditions can be met in the vicinity of the origin, by appropriately choosing fluxes. The SUSY conditions (5.5) can be solved for ψ and τ to get

$$\psi^2 = -\frac{c_0}{c_2}\frac{\tilde{f}\cdot\tilde{p}_0}{\tilde{f}\cdot\tilde{p}_2}\,, \qquad \tau = \frac{\tilde{f}\cdot\tilde{p}_2}{\tilde{h}\cdot\tilde{p}_2}\,. \tag{5.21}$$

   They also impose restrictions on the fluxes

$$(\tilde{f}\cdot\tilde{p}_2)(\tilde{h}\cdot\tilde{p}_0) - (\tilde{f}\cdot\tilde{p}_0)(\tilde{h}\cdot\tilde{p}_2) = 0\,. \tag{5.22}$$

   A straightforward calculation shows that the conditions on fluxes (5.22) can be rewritten as

$$\begin{aligned} \tilde{f}_1\tilde{h}_3 - \tilde{f}_3\tilde{h}_1 + \tilde{f}_4\tilde{h}_2 - \tilde{f}_2\tilde{h}_4 &= 0\,, \\ \tilde{f}_1(\tilde{h}_4 - \tilde{h}_2) + \tilde{f}_2(\tilde{h}_1 + \tilde{h}_3) - \tilde{f}_3(\tilde{h}_2 + \tilde{h}_4) + \tilde{f}_4(\tilde{h}_3 - \tilde{h}_1) &= 0\,. \end{aligned} \tag{5.23}$$

   For ease of analysis we will continue to consider fluxes which meet the conditions (5.16) and (5.19). Equation (5.23) then gives the condition

$$\tilde{h}_4 = -\frac{2\tilde{h}_1\tilde{f}_1}{\tilde{f}_2}\,. \tag{5.24}$$

   We will furnish concrete examples below to show that (5.16), (5.19) and (5.24) can be satisfied for appropriate integer quantized fluxes.

   Once the restrictions on the flux are met, a solution exists to this order. Using (5.16) and (5.19) we see that ψ and τ are given by

$$\psi^2 = -i\frac{c_0}{c_2}\left[\frac{\tilde{f}_1 + \frac{1}{\sqrt{2}}\tilde{f}_2}{\tilde{f}_1 - \frac{1}{\sqrt{2}}\tilde{f}_2}\right], \tag{5.25}$$

$$\tau = -\frac{i}{\sqrt{2}}\frac{\tilde{f}_2}{\tilde{h}_1}\,. \tag{5.26}$$

For the $\mathcal{O}(\psi^2)$ analysis to be valid the resulting value of ψ should satisfy $|\psi| \ll 1$. We see that this can be arranged if $\tilde{f}_1 \simeq -\frac{1}{\sqrt{2}}\tilde{f}_2$, as would be expected from our discussion of a susy vacuum at $\psi = 0$.[4]

We also note that the total three brane charge carried by the flux, satisfying the conditions (5.16), (5.19) and (5.24), is given by

$$N_{\text{flux}} = -2\frac{\tilde{h}_1}{\tilde{f}_2}(\tilde{f}_2^2 - 4\tilde{f}_1\tilde{f}_2 + 2\tilde{f}_1^2)\,. \tag{5.27}$$

As an explicit example we take

$$\begin{aligned} f &= (-28, 24, 7, -24)\,, \\ h &= (-34, 41, 12, -17)\,. \end{aligned} \tag{5.28}$$

The resulting values for $\tilde{f}$ and $\tilde{h}$ are

$$\begin{aligned} \tilde{f} &= (7, -10, 7, 0)\,, \\ \tilde{h} &= (5, 0, -5, 7)\,, \end{aligned} \tag{5.29}$$

and satisfy the conditions (5.16), (5.19) and (5.24). Also, $\tilde{f}_1/\tilde{f}_2 = -7/10$, so that $-\sqrt{2}\tilde{f}_1/\tilde{f}_2 - 1 \simeq -0.01$, which is quite small. As a result we expect that $|\psi| \ll 1$ in this example. Indeed, inserting the values of c_0 and c_2 from section 3.2, we obtain from (5.25)

$$\psi^2 \simeq 6.47(1-i) \times 10^{-3}\,, \tag{5.30}$$

which is quite small. The resulting value of the dialton-axion is $\tau = \sqrt{2}i$.

Also we note that for this example $N_{\text{flux}} = 478$ which is much less than the maximum allowed value $\chi(X_A)/24 = 972$.

There is one subtlety which we have not fully analyzed in this model. On an orientifold, due to possible "half cycles" [7], sometimes the fluxes f and h need to be even integer (though often in cases where the subtlety arises, the odd fluxes can be rendered consistent by turning on fractional fluxes at orientifold planes [7]). It could be that this subtlety makes the choice of fluxes (5.28) inconsistent.

However one can easily find other examples which involve only even flux integers. For example one can take

$$f = (24, -20, -6, 20) \qquad \text{and} \qquad h = (28, -34, -10, 14)\,. \tag{5.31}$$

The resulting values for $\tilde{f}$ and $\tilde{h}$ are

$$\tilde{f} = (-6, 8, -6, 0) \qquad \text{and} \qquad \tilde{h} = (-4, 0, 4, -6)\,. \tag{5.32}$$

[4]Note that if $\tilde{f}_1 \simeq -\frac{1}{\sqrt{2}}\tilde{f}_2$, then from (5.24), $\frac{\tilde{h}_1}{\tilde{h}_4} \simeq \frac{1}{\sqrt{2}}$, so (5.20) is also approximately met.

Again, $\tilde{f}_1/\tilde{f}_2 = -3/4$ is close to $-1/\sqrt{2}$. In this case ψ turns out to be

$$\psi^2 \simeq 0.038(i-1)\,, \qquad |\psi|^2 \simeq 0.053\,, \tag{5.33}$$

again much less than one, and the resulting value of the dilaton-axion is $\tau = \sqrt{2}i$. Also, $N_{\rm flux} = 328$ which is less than the total allowed value, $\chi(X_A)/24 = 972$, in this example.

To summarize, we see that appropriate fluxes can be turned on in model A to meet the conditions of supersymmetry up to $\mathcal{O}(\psi^2)$. The resulting vacuum lies at $|\psi| \ll 1$, so we expect the $\mathcal{O}(\psi^2)$ approximation is good in determining the location of the vacuum.

3. Supersymmetry Breaking at $\mathcal{O}(\psi^4)$.

The equations imposing supersymmetry, eq. (3.9), are overdetermined. Therefore one expects that at higher orders supersymmetry will be broken in this model. Since ψ is small in the solution above, and to $\mathcal{O}(\psi^2)$ we have a SUSY solution, we expect the resulting supersymmetry breaking to be somewhat suppressed. By carrying out the analysis to $\mathcal{O}(\psi^4)$ in this section we will find this is true. Our analysis will also ensure that that the solution found above extends in perturbation theory to a solution of the equations of motion in higher orders.

We will sketch out some of the steps here, more details are furnished in appendix A. From eq. (A.2) the superpotential is given by

$$W_A = c_0(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2\psi^2 + c_4(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_4\psi^4 + \mathcal{O}(\psi^6)\,. \tag{5.34}$$

A solution to the classical equations of motion must meet the conditions

$$D_\tau W_A \equiv \partial_\tau W_A + \partial_\tau K W_A = 0\,, \tag{5.35}$$

and

$$D_\psi W_A \equiv \partial_\psi W_A + \partial_\psi K W_A = 0\,, \tag{5.36}$$

where K is the Kähler potential. Eq. (A.6) in appendix A tells us that it is given by

$$K = -\log\left\{-i(\tau-\bar{\tau})\right\} - \log\left\{2[(2+\sqrt{2})|c_0|^2 + (-2+\sqrt{2})|c_2\psi^2|^2] + \mathcal{O}(\psi^6)\right\}. \tag{5.37}$$

Eq. (5.35) and (5.36) then take the form

$$\begin{aligned} D_\psi W_A &= 2\psi\Bigg[c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2 + 2c_4(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_4\psi^2 + \left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)\left\{\frac{|\,c_2\,|}{|\,c_0\,|}\right\}^2 \times \\ &\qquad \times\,\bar{\psi}^2\left(c_0(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2\psi^2\right) + \mathcal{O}(\psi^6)\Bigg]\,, \\ D_\tau W_A &= -\Bigg[\tilde{h}\cdot(c_0\tilde{p}_0 + \psi^2 c_2\tilde{p}_2 + \psi^4 c_4\tilde{p}_4) + \frac{1}{\tau-\bar{\tau}}\times \\ &\qquad \times\left\{c_0(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2\psi^2 + c_4(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_4\psi^4\right\} + \mathcal{O}(\psi^6)\Bigg]. \end{aligned} \tag{5.38}$$

Let ψ_0 and τ_0 denote the SUSY preserving solutions obtained in the previous subsection by working up to $\mathcal{O}(\psi^2)$. They satisfy eq. (5.5). A consistent solution to (5.38) can then be obtained by taking ψ and τ to be of the form

$$\begin{aligned} \psi &= \psi_0 + \alpha_\psi \psi_0^3 + \mathcal{O}(\psi_0^5)\,, \\ \tau &= \tau_0 + \alpha_\tau \psi_0^2 + \mathcal{O}(\psi_0^4)\,. \end{aligned} \tag{5.39}$$

From $D_\psi W_A = 0$ we get

$$\psi_0^2 \left(-\alpha_\tau c_2 \tilde{h} \cdot \tilde{p}_2 + 2c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \right) + \mathcal{O}(\psi_0^4) = 0\,, \tag{5.40}$$

solving which we find

$$\alpha_\tau = \frac{2c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4}{c_2 \tilde{h} \cdot \tilde{p}_2}\,. \tag{5.41}$$

Similarly, $D_\tau W_A = 0$ gives

$$\psi_0^4 \left(2c_2 \tilde{h} \cdot \tilde{p}_2 \alpha_\psi + c_4 \left\{ \tilde{h} \cdot \tilde{p}_4 + \frac{1}{\tau_0 - \bar{\tau}_0} (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \right\} \right) + \mathcal{O}(\psi_0^6) = 0\,, \tag{5.42}$$

and hence

$$\alpha_\psi = -\frac{c_4}{2c_2 \tilde{h} \cdot \tilde{p}_2} \left\{ \tilde{h} \cdot \tilde{p}_4 + \frac{1}{\tau_0 - \bar{\tau}_0} (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \right\}. \tag{5.43}$$

Let us now evaluate the $\mathcal{O}(\psi^4)$ corrections for the two examples, (5.28) and (5.31), considered above. Substituting the values of ψ_0, τ_0, $\tilde{f}$, $\tilde{h}$, c_i and $\tilde{p}_i$ we find that the resulting values of α_ψ and α_τ are very close in the two examples, (5.28) and (5.31). It turns out that $\alpha_\psi = 0$ and

$$\alpha_\tau \simeq 1.073(1 - i)\,. \tag{5.44}$$

We can now determine the scale of supersymmetry breaking. The superpotential is

$$W_A = c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^4 + \mathcal{O}(\psi_0^6)\,. \tag{5.45}$$

For the choice of fluxes (5.28), this gives

$$e^{K/2} |W| \simeq \alpha' 4.43 \times 10^{-3}\,, \tag{5.46}$$

where we have restored a factor of $(2\pi)^2 \alpha'$ in the relative normalisation between W, (3.9), and W_A, (5.2). Thus the scale of supersymmetry breaking is indeed quite small compared to the string scale.

For the example (5.31), when all the fluxes are even integers, we get

$$e^{K/2} |W| \simeq \alpha' 0.125\,, \tag{5.47}$$

so the scale of breaking is only moderately smaller than the string scale.

Let us end this section with a few more comments. The KKLT construction [18] involves a small parameter W_0. This is the value of the tree level superpotential in the effective theory obtained after integrating out the complex structure moduli and the dilaton. It is easy to see that $|W_0|$ in KKLT is exactly the same as $e^{K/2}|W|$ calculated in (5.46) (note for this purpose that K in (5.46) refers to the Kähler potential of the complex structure moduli and the dilaton-axion fields alone, (5.37), not the volume modulus). As was mentioned at the beginning of this section we allowed only four of the fluxes to be turned on in model A. It is encouraging to note that even with this limited number a modestly small value of $|W_0|$ has been obtained in a construction which meets several of the other requirements of the KKLT construction as well.

Finally, we cannot refrain from mentioning one curiosity. For both examples, (5.28) and (5.31), we have found an alternate choice of flux which yields a very similar vacuum. (5.28) is paired with the choice $f = (-20, 17, 5, -17)$, $h = (-48, 58, 17, -24)$, and (5.31) with $f = (16, -14, -4, 14)$, $h = (40, -48, -14, 20)$. The dilaton expectation value and $N_{\rm flux}$ is the same in each pair. The supersymmetry breaking scale is quite similar too, differing by about one percent in each pair. And the value of ψ_0^2 is similarly close, up to a sign. To the best of our knowledge, this is not the consequence of any known duality symmetry.

5.2 Model B

In this subsection we present some nonsupersymmetric solutions in model B with nonvanishing W (which is not particularly small). Let us restrict ourselves to the point $\psi = \phi = 0$ and look for vacua satisfying

$$W \neq 0\,, \qquad D_\tau W = 0\,, \qquad \frac{1}{\psi} D_\psi W = 0\,, \qquad D_\phi W = 0\,. \tag{5.48}$$

This gives the following conditions on fluxes

$$\begin{aligned} (f - \tau h)\cdot(p_{01} + \alpha p_{02}) &\neq 0\,, \qquad & (f - \overline{\tau} h)\cdot(p_{01} + \alpha p_{02}) &= 0\,, \\ (f - \tau h)\cdot(p_{21} + i p_{22}) &= 0\,, \qquad & (f - \tau h)\cdot(p_{01} - \alpha p_{02}) &= 0\,, \end{aligned} \tag{5.49}$$

where $\alpha = \exp(\pi i/6)$. The third condition may be solved by putting

$$f = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ -4f_1 + 2f_4 \\ -2f_1 - f_2 \end{pmatrix}, \qquad h = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ -4h_1 + 2h_4 \\ -2h_1 - h_2 \end{pmatrix}. \tag{5.50}$$

The second and fourth conditions may be used to fix dilaton-axion to the value

$$\tau = \frac{f\cdot(p_{01} - \alpha p_{02})}{h\cdot(p_{01} - \alpha p_{02})}\,. \tag{5.51}$$

This solution for τ will be consistent with both conditions if

$$\frac{f\cdot(p_{01} - \alpha p_{02})}{h\cdot(p_{01} - \alpha p_{02})} = \frac{f\cdot(p_{01}^\dagger + \alpha^\dagger p_{02}^\dagger)}{h\cdot(p_{01}^\dagger + \alpha^\dagger p_{02}^\dagger)}\,. \tag{5.52}$$

Now for simplicity, we consider the case when the numerator and denominator are separately equal. This finally gives the following two parameter families of fluxes

$$f = \begin{pmatrix} -2f_2 \\ f_2 \\ -5f_2 - 2f_4 \\ f_4 \\ 8f_2 + 2f_4 \\ 3f_2 \end{pmatrix}, \qquad h = \begin{pmatrix} -2h_2 \\ h_2 \\ -5h_2 - 2h_4 \\ h_4 \\ 8h_2 + 2h_4 \\ 3h_2 \end{pmatrix}. \tag{5.53}$$

The dilaton-axion is equal to

$$\tau = \frac{2f_2 + f_4 + if_2}{2h_2 + h_4 + ih_2}. \tag{5.54}$$

The superpotential (evaluated in the vacuum) in this case is equal to

$$W = N_w(f - \tau h) \cdot (p_{01} + \alpha p_{02}) = \frac{iN_w N_{\rm flux}}{2h_2 + h_4 + ih_2}, \tag{5.55}$$

where $N_w = (2\pi)^2 c_0 u_{-\frac{1}{6}}(0)\alpha'$ and $N_{\rm flux} = 6(f_2h_4 - h_2f_4)$. It can be expressed in terms of $\mathrm{Im}(\tau)$ and the h fluxes

$$W = 6iN_w \mathrm{Im}(\tau)(2h_2 + h_4 - ih_2). \tag{5.56}$$

Hence,

$$e^{K/2}|W| = (2\pi)^2\alpha' \sqrt{3\,\mathrm{Im}(\tau)}\,|2h_2 + h_4 - ih_2|. \tag{5.57}$$

For instance, let us choose even fluxes

$$\begin{aligned} f &= (8, -4, 4, 8, -16, -12), \\ h &= (0, 0, 4, -2, -4, 0). \end{aligned} \tag{5.58}$$

This gives $\tau = 2i$, $N_{\rm flux} = 48$, $h_2 = 0$, $h_4 = -2$ and yields the following

$$e^{K/2}|W| = (2\pi)^2\alpha' 2\sqrt{6} \simeq \alpha'\,193, \tag{5.59}$$

which is actually $10^3 - 10^5$ larger in comparison with the nonsupersymmetric solutions in model A.

6. Discussion

In this brief paper, we have seen examples of two interesting phenomena: the IIB flux equations on some Calabi-Yau threefolds admit supersymmetric solutions even in the leading approximation (despite the fact that the no-scale SUSY equations are overdetermined), and one can find nonsupersymmetric solutions with relatively small W (even by turning on only a handful of fluxes). Both of these results provide further motivation to develop models of particle physics [5] and [13]–[15] and cosmology [18, 19, 36] (including D-brane inflation [37, 38]) in this general framework.

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There are a couple of obvious directions for further work. Our results have been exploratory in nature, only exhibiting a handful of solutions in examples which admit easy F-theory lifts. Any more general results on the space of solutions in a given example could complement the "generic" analysis of [20] with detailed specific information, presently only available in the simple cases of $T^6/\mathbb{Z}_2$ and $K3 \times T^2/\mathbb{Z}_2$ compactifications.

In addition, the solutions described here provide a further step towards making completely explicit models of the proposals [18, 19] for realizing de Sitter vacua in string theory (see also [39] for earlier proposals in noncritical string theory). Indeed, the F-theory models on X_A and X_B admit stacks of D7 branes which could (when appropriately stabilized) yield non-abelian gauge groups and gaugino condensates. It is plausible that more work along these lines could lead to a very explicit realization of the proposal of [18], though of course one would very likely have to turn on more generic fluxes than the small subset we have used here.

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A. More details on model A

In this appendix we obtain the nonsupersymmetric solutions for model A by solving the equations up to $\mathcal{O}(\psi^4)$.

For convenience, we will first rewrite the superpotential (5.2)

$$W_A = f \cdot \Pi_A - \tau h \cdot \Pi_A \tag{A.1}$$

in a different form. Using the eqs. (5.6), (3.16) and (3.15), in the above, we obtain

$$\begin{aligned} W_A &= \tilde{f} \cdot m_A^{-1} \cdot \Pi_A - \tau \tilde{h} \cdot m_A^{-1} \cdot \Pi_A \\ &= c_0(\tilde{f} - \tau\tilde{h}) \cdot \tilde{p}_0 + c_2(\tilde{f} - \tau\tilde{h}) \cdot \tilde{p}_2\psi^2 + c_4(\tilde{f} - \tau\tilde{h}) \cdot \tilde{p}_4\psi^4 + \mathcal{O}(\psi^6)\,. \end{aligned} \tag{A.2}$$

Similarly, using the periods (3.15) the Kähler potential (3.8) for model A

$$K = -\ln(-i(\tau - \bar{\tau})) - \ln(-i\Pi_A^\dagger \cdot \Sigma \cdot \Pi_A) \tag{A.3}$$

can be expressed as

$$\begin{aligned} K = &- \ln(-i(\tau - \bar{\tau})) - \\ &- \ln\Big(-i(c_0p_0 + c_2p_2\psi^2 + c_4p_4\psi^4)^\dagger \cdot \Sigma \cdot (c_0p_0 + c_2p_2\psi^2 + c_4p_4\psi^4)\Big)\,. \end{aligned} \tag{A.4}$$

It is straightforward to check that

$$
\begin{aligned}
p_0^\dagger \cdot \Sigma \cdot p_2 &= p_0^\dagger \cdot \Sigma \cdot p_4 = 0\,, \\
p_0^\dagger \cdot \Sigma \cdot p_0 &= 2i(2+\sqrt{2})\,, \\
p_2^\dagger \cdot \Sigma \cdot p_2 &= 2i(-2+\sqrt{2})\,.
\end{aligned} \tag{A.5}
$$

Clearly, eq. (A.4) becomes

$$
K = -\ln(-i(\tau-\bar{\tau})) - \ln\{2[(2+\sqrt{2})|c_0|^2 + (-2+\sqrt{2})|c_2\psi^2|^2] + \mathcal{O}(\psi^6)\}. \tag{A.6}
$$

Taking the partial derivatives with respect to ψ and τ we get

$$
\begin{aligned}
\partial_\psi K &= -\frac{2\psi(-2+\sqrt{2})|c_2|^2\bar{\psi}^2}{(2+\sqrt{2})|c_0|^2 + (-2+\sqrt{2})|c_2\psi^2|^2} \\
&= 2\psi\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)\left\{\frac{|\,c_2\,|}{|\,c_0\,|}\right\}^2 \bar{\psi}^2 + \mathcal{O}(\psi^4)\,,
\end{aligned} \tag{A.7}
$$

and

$$
\partial_\tau K = -\frac{1}{\tau-\bar{\tau}}\,. \tag{A.8}
$$

We can now evaluate the covariant derivatives (5.35) and (5.36) as follows:

$$
\begin{aligned}
D_\psi W_A = 2\psi\Bigg[& c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2 + 2c_4(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_4\psi^2 + \left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)\left\{\frac{|\,c_2\,|}{|\,c_0\,|}\right\}^2 \bar{\psi}^2 \times \\
&\times\Big(c_0(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2\psi^2\Big) + \mathcal{O}(\psi^6)\Bigg]
\end{aligned} \tag{A.9}
$$

and

$$
\begin{aligned}
D_\tau W_A = -\Bigg[&\tilde{h}\cdot(c_0\tilde{p}_0 + \psi^2 c_2\tilde{p}_2 + \psi^4 c_4\tilde{p}_4) + \frac{1}{\tau-\bar{\tau}}\times \\
&\times\Big\{c_0(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_2\psi^2 + c_4(\tilde{f}-\tau\tilde{h})\cdot\tilde{p}_4\psi^4\Big\} + \mathcal{O}(\psi^6)\Bigg].
\end{aligned} \tag{A.10}
$$

Let ψ_0 and τ_0 be the SUSY preserving solutions obtained from eq. (5.5). To find solutions up to $\mathcal{O}(\psi^4)$, we observe that the $D_\tau W_A = 0$ condition implies $\psi \sim \psi_0 + \mathcal{O}(\psi_0^3)$ and similarly $D_\tau W_A = 0$ implies $\tau \sim \tau_0 + \mathcal{O}(\psi_0^2)$. Thus we take the following ansatz for ψ and τ:

$$
\begin{aligned}
\psi &= \psi_0 + \alpha_\psi\psi_0^3 + \mathcal{O}(\psi_0^5)\,, \\
\tau &= \tau_0 + \alpha_\tau\psi_0^2 + \mathcal{O}(\psi_0^4)\,.
\end{aligned} \tag{A.11}
$$

Putting these in the superpotential (A.2) results in

$$
\begin{aligned}
W_A = {}& c_0(\tilde{f}-\tau_0\tilde{h})\cdot\tilde{p}_0 + c_2(\tilde{f}-\tau_0\tilde{h})\cdot\tilde{p}_2\psi_0^2 - \alpha_\tau\tilde{h}\cdot(c_0p_0 + \psi_0^2c_2p_2)\psi_0^2 + \\
&+ 2\alpha_\psi c_2(\tilde{f}-\tau_0\tilde{h})\cdot\tilde{p}_2\psi_0^4 + c_4(\tilde{f}-\tau_0\tilde{h})\cdot\tilde{p}_4\psi_0^4 + \mathcal{O}(\psi_0^6)\,.
\end{aligned} \tag{A.12}
$$

Most of the terms in the r.h.s. of the above equation vanish and finally W_A becomes

$$W_A = c_4(\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^4 + \mathcal{O}(\psi_0^6)\,. \tag{A.13}$$

This is expected, since up to quadratic order in ψ both $W_A(\psi_0, \tau_0)$ and $dW_A(\psi_0, \tau_0)$ vanish. This means that $W_A(\psi_0, \tau_0) \sim \mathcal{O}(\psi_0^4)$ as we find in the above.

Since $\partial_\psi K \sim \mathcal{O}(\psi_0^3)$ and $W_A \sim \mathcal{O}(\psi_0^4)$ the $(\partial_\psi K) W_A$ term does not contribute terms up to $\mathcal{O}(\psi^4)$ in $D_\psi W_A$, whereas $\partial_\tau K \sim 1 + \mathcal{O}(\psi_0^2)$ and hence both the terms in $D_\tau W_A$ are significant. Using (A.11), we can now easily expand the r.h.s. of (A.9) and (A.10). Consider first

$$D_\psi W_A = 2(\psi_0 + \alpha_\psi \psi_0^3) \left[c_2(\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_2 - \alpha_\tau \psi_0^2 c_2 \tilde{h} \cdot \tilde{p}_2 + 2 c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^2 + \mathcal{O}(\psi_0^4) \right]. \tag{A.14}$$

Again, using eq. (5.5) we can simplify it:

$$D_\psi W_A = 2\psi_0 \left[-\alpha_\tau \psi_0^2 c_2 \tilde{h} \cdot \tilde{p}_2 + 2 c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^2 + \mathcal{O}(\psi_0^4) \right]. \tag{A.15}$$

From $D_\psi W_A = 0$ we then find

$$\alpha_\tau = \frac{2 c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4}{c_2 \tilde{h} \cdot \tilde{p}_2} + \mathcal{O}(\psi_0^2)\,. \tag{A.16}$$

Similarly, we consider

$$\begin{aligned} D_\tau W_A = -\Big[& \tilde{h} \cdot (c_0 \tilde{p}_0 + \psi_0^2 c_2 \tilde{p}_2 + 2\alpha_\psi \psi_0^4 c_2 \tilde{p}_2 + \psi_0^4 c_4 \tilde{p}_4) + \frac{1}{\tau_0 - \bar{\tau}_0} \times \\ & \times \{ c_0 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_0 + c_2 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_2 \psi_0^2 - \alpha_\tau \tilde{h} \cdot (c_0 p_0 + \psi_0^2 c_2 p_2) \psi_0^2 + \\ & + 2\alpha_\psi c_2 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_2 \psi_0^4 + c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^4 \} + \mathcal{O}(\psi_0^6) \Big], \end{aligned} \tag{A.17}$$

which, upon using eq. (5.5), reduces to

$$D_\tau W_A = -\left[h \cdot (2\alpha_\psi c_2 \tilde{p}_2 + c_4 \tilde{p}_4) \psi_0^4 + \frac{1}{\tau_0 - \bar{\tau}_0} c_4 (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \psi_0^4 + \mathcal{O}(\psi_0^6) \right]. \tag{A.18}$$

Solving $D_\tau W_A = 0$ yields

$$\alpha_\psi = -\frac{c_4}{2 c_2 \tilde{h} \cdot \tilde{p}_2} \left\{ \tilde{h} \cdot \tilde{p}_4 + \frac{1}{\tau_0 - \bar{\tau}_0} (\tilde{f} - \tau_0 \tilde{h}) \cdot \tilde{p}_4 \right\}. \tag{A.19}$$

B. More details on model B

In this appendix we provide a monodromy group basis for the hypersurface in $WP^4_{1,1,2,2,6}$ in terms of three matrices denoted by A, T and B in the symplectic (large complex structure) basis and a, t and b in the Picard-Fuchs basis. The former were computed in [29], while the latter appear in a very similar model in [26]. The two bases are related by the transformation

$$A = m_B \cdot a \cdot m_B^{-1}\,, \qquad T = m_B \cdot t \cdot m_B^{-1}\,, \qquad B = m_B \cdot b \cdot m_B^{-1}\,, \tag{B.1}$$

where the matrix m_B is defined in (3.21). These monodromies are obtained by loops in the two parameter moduli space around the $\mathbb{Z}_{12}$ identified point $\psi = 0$, the conifold singularity (which is $864\psi^6 + \phi = \pm 1$), and the strong coupling singularity ($\phi^2 = 1$).

B.1 Monodromy group in symplectic (large complex structure) basis

In this subsection we reproduce the monodromy matrices given in [29] in the symplectic (large complex structure) basis. They are

$$A = \begin{pmatrix} -1 & 01-200 & \\ 0 & 1 & 0020 \\ -1 & 1 & -1-121 \\ 1 & 0 & 0100 \\ -1 & 0 & 0-111 \\ 1 & 0 & 010-1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 00000 & \\ 0 & 1 & 0000 \\ 0 & 0 & 1000 \\ -1 & 0 & 0100 \\ 0 & 0 & 0010 \\ 0 & 0 & 0001 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & -12-1-21 & \\ 0 & 1 & 020-2 \\ 0 & 1 & -112-1 \\ 0 & 0 & 0100 \\ 0 & 0 & 0-111 \\ 0 & 0 & 020-1 \end{pmatrix}. \tag{B.2}$$

Here $B = (T_2 A T)^{-1}$, where T_2 is given in [29].

B.2 Monodromy group in Picard-Fuchs basis

In this subsection we compute the monodromy matrices in the Picard-Fuchs basis explicitly. The monodromy around $\psi = 0$ is the simplest and is given by

$$(\psi, \phi) \to (\alpha\psi, -\phi)\,. \tag{B.3}$$

The explicit expression for the period vector which follows from (3.19) yields

$$a = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{B.4}$$

Knowing this monodromy matrix a and the matrix A from (B.2), one can compute m_B in (3.21) using the relation (B.1).

Now knowing m_B and T and B from (B.2), one can compute t and b in the Picard-Fuchs basis. They are given below

$$t = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ -2 & 2 & 0 & 1 & 0 & 0 \\ 2 & -2 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{B.5}$$

$$b = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 1 \\ -1 & 1 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & 2 & -2 \\ 0 & 0 & -1 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{B.6}$$

Next we check explicitly that (B.5) is indeed the monodromy around the conifold point. To this end, following the analysis of [26] for the hypersurface in $WP^8_{1,1,2,2,2}$, let us rewrite the periods in the following form

$$\begin{aligned} w_{2j}(\psi,\phi) &= -\frac{1}{6\pi^3}\sum_{r=1}^{6}(-1)^r \sin^2(\frac{\pi r}{6})\sin(\frac{\pi r}{2})\alpha^{2jr}\xi^r_{2j}, \\ \xi^r_{2j} &= \sum_{n=1}^{\infty}\frac{\Gamma^3(n+\frac{r}{6})\Gamma(3(n+\frac{r}{6}))}{\Gamma(6(n+\frac{r}{6}))}(-1)^n(12\psi)^{6n+r}u_{-(n+\frac{r}{6})}(\phi)\,; \\ w_{2j+1}(\psi,\phi) &= -\frac{1}{6\pi^3}\sum_{r=1}^{6}(-1)^r \sin^2(\frac{\pi r}{6})\sin(\frac{\pi r}{2})\alpha^{(2j+1)r}\xi^r_{2j+1}\,, \\ \xi^r_{2j+1} &= \sum_{n=1}^{\infty}\frac{\Gamma^3(n+\frac{r}{6})\Gamma(3(n+\frac{r}{6}))}{\Gamma(6(n+\frac{r}{6}))}(12\psi)^{6n+r}u_{-(n+\frac{r}{6})}(-\phi)\,; \end{aligned} \tag{B.7}$$

where $j=0,1,2$ and $\alpha=\exp(\frac{\pi i}{6})$. Next we follow the analysis of [24] and [26] and find

$$\frac{d^2w_i(\psi,0)}{d\psi^2}\sim \text{const}\frac{c_i}{1-864\psi^6}\,,$$

for $864\psi^6$ in vicinity of 1 and $c_i=(1,1,-1,-2,2,1)$, for $i=0,\ldots,5$. Here one gets this using the Stirling formula for the expansion of Γ-functions and the following result of [26]

$$u_\nu(0)=\frac{2^\nu\sqrt{\pi}\exp(\frac{\pi i\nu}{2})}{\Gamma(1+\frac{\nu}{2})\Gamma(\frac{1}{2}-\frac{\nu}{2})}\,. \tag{B.8}$$

The resulting monodromy t around the conifold point is given by

$$w_j\to w_j+c_j(w_0-w_1)\,, \tag{B.9}$$

which exactly coincides with the matrix (B.5).

References

[1] J. Polchinski and A. Strominger, *New vacua for type-II string theory*, *Phys. Lett.* **B 388** (1996) 736 [hep-th/9510227];
K. Becker and M. Becker, *M-theory on eight-manifolds*, *Nucl. Phys.* **B 477** (1996) 155 [hep-th/9605053];
J. Michelson, *Compactifications of type-IIB strings to four dimensions with non-trivial classical potential*, *Nucl. Phys.* **B 495** (1997) 127 [hep-th/9610151];
K. Becker and M. Becker, *Supersymmetry breaking, M-theory and fluxes*, *J. High Energy Phys.* **07** (2001) 038 [hep-th/0107044];
M. Haack and J. Louis, *M-theory compactified on Calabi-Yau fourfolds with background flux*, *Phys. Lett.* **B 507** (2001) 296 [hep-th/0103068];
J. Louis and A. Micu, *Type II theories compactified on Calabi-Yau threefolds in the presence of background fluxes*, *Nucl. Phys.* **B 635** (2002) 395 [hep-th/0202168].

[2] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [hep-th/9906070].

[3] K. Dasgupta, G. Rajesh and S. Sethi, *M-theory, orientifolds and G-flux*, *J. High Energy Phys.* **08** (1999) 023 [hep-th/9908088];
B.R. Greene, K. Schalm and G. Shiu, *Warped compactifications in M and F theory*, *Nucl. Phys.* **B 584** (2000) 480 [hep-th/0004103].

[4] G. Curio, A. Klemm, D. Lüst and S. Theisen, *On the vacuum structure of type-II string compactifications on Calabi-Yau spaces with H-fluxes*, *Nucl. Phys.* **B 609** (2001) 3 [hep-th/0012213].

[5] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[6] S. Kachru, M.B. Schulz and S. Trivedi, *Moduli stabilization from fluxes in a simple IIB orientifold*, *J. High Energy Phys.* **10** (2003) 007 [hep-th/0201028].

[7] A.R. Frey and J. Polchinski, $N = 3$ *warped compactifications*, *Phys. Rev.* **D 65** (2002) 126009 [hep-th/0201029].

[8] P.K. Tripathy and S.P. Trivedi, *Compactification with flux on K3 and tori*, *J. High Energy Phys.* **03** (2003) 028 [hep-th/0301139].

[9] M. Bianchi, G. Pradisi and A. Sagnotti, *Toroidal compactification and symmetry breaking in open string theories*, *Nucl. Phys.* **B 376** (1992) 365;
S. Chaudhuri, G. Hockney and J.D. Lykken, *Maximally supersymmetric string theories in* $D < 10$, *Phys. Rev. Lett.* **75** (1995) 2264 [hep-th/9505054];
S. Chaudhuri and D.A. Lowe, *Type iia heterotic duals with maximal supersymmetry*, *Nucl. Phys.* **B 459** (1996) 113 [hep-th/9508144]; *Monstrous string-string duality*, *Nucl. Phys.* **B 469** (1996) 21 [hep-th/9512226].

[10] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152];
P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198].

[11] H. Verlinde, *Holography and compactification*, *Nucl. Phys.* **B 580** (2000) 264 [hep-th/9906182];

JHEP04(2004)003

C.S. Chan, P.L. Paul and H. Verlinde, *A note on warped string compactification*, *Nucl. Phys.* **B 581** (2000) 156 [hep-th/0003236].

[12] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [hep-th/0007191].

[13] J.F.G. Cascales, M.P. Garcia del Moral, F. Quevedo and A.M. Uranga, *Realistic D-brane models on warped throats: fluxes, hierarchies and moduli stabilization*, *J. High Energy Phys.* **02** (2004) 031 [hep-th/0312051];
J.F.G. Cascales and A.M. Uranga, *Chiral 4d string vacua with D-branes and moduli stabilization*, hep-th/0311250;
Chiral 4d $N = 1$ string vacua with D-branes and NS-NS and RR fluxes, *J. High Energy Phys.* **05** (2003) 011 [hep-th/0303024].

[14] R. Blumenhagen, D. Lüst and T.R. Taylor, *Moduli stabilization in chiral type-IIB orientifold models with fluxes*, *Nucl. Phys.* **B 663** (2003) 319 [hep-th/0303016].

[15] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, hep-th/0311241;
M. Grana, *MSSM parameters from supergravity backgrounds*, *Phys. Rev.* **D 67** (2003) 066006 [hep-th/0209200].

[16] A.R. Frey, *Warped strings: self-dual flux and contemporary compactifications*, hep-th/0308156.

[17] R. D'Auria, S. Ferrara and S. Vaula, *$N = 4$ gauged supergravity and a IIB orientifold with fluxes*, *New J. Phys.* **4** (2002) 71 [hep-th/0206241];
S. Ferrara and M. Porrati, *$N = 1$ no-scale supergravity from IIB orientifolds*, *Phys. Lett.* **B 545** (2002) 411 [hep-th/0207135];
R. D'Auria, S. Ferrara, M.A. Lledó and S. Vaula, *No-scale $N = 4$ supergravity coupled to Yang-Mills: the scalar potential and super Higgs effect*, *Phys. Lett.* **B 557** (2003) 278 [hep-th/0211027];
R. D'Auria, S. Ferrara, F. Gargiulo, M. Trigiante and S. Vaula, *$N = 4$ supergravity lagrangian for type-IIB on $T^6/\mathbb{Z}_2$ in presence of fluxes and D3-branes*, *J. High Energy Phys.* **06** (2003) 045 [hep-th/0303049];
L. Andrianopoli, S. Ferrara and M. Trigiante, *Fluxes, supersymmetry breaking and gauged supergravity*, hep-th/0307139;
B. de Wit, H. Samtleben and M. Trigiante, *Maximal supergravity from IIB flux compactifications*, *Phys. Lett.* **B 583** (2004) 338 [hep-th/0311224].

[18] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[19] C. Escoda, M. Gomez-Reino and F. Quevedo, *Saltatory de Sitter string vacua*, *J. High Energy Phys.* **11** (2003) 065 [hep-th/0307160];
C.P. Burgess, R. Kallosh and F. Quevedo, *de Sitter string vacua from supersymmetric d-terms*, *J. High Energy Phys.* **10** (2003) 056 [hep-th/0309187].

[20] S. Ashok and M.R. Douglas, *Counting flux vacua*, *J. High Energy Phys.* **01** (2004) 060 [hep-th/0307049];
M.R. Douglas, *The statistics of string/M theory vacua*, *J. High Energy Phys.* **05** (2003) 046 [hep-th/0303194].

[21] D.R. Morrison, *Picard-Fuchs equations and mirror maps for hypersurfaces*, hep-th/9111025.

[22] A. Klemm and S. Theisen, *Considerations of one modulus Calabi-Yau compactifications: Picard-Fuchs equations, Kähler potentials and mirror maps*, *Nucl. Phys.* **B 389** (1993) 153 [hep-th/9205041].

[23] A. Font, *Periods and duality symmetries in Calabi-Yau compactifications*, *Nucl. Phys.* **B 391** (1993) 358 [hep-th/9203084].

[24] P. Candelas, X.C. De La Ossa, P.S. Green and L. Parkes, *A pair of Calabi-Yau manifolds as an exactly soluble superconformal theory*, *Nucl. Phys.* **B 359** (1991) 21.

[25] B.R. Greene and M.R. Plesser, *Duality in Calabi-Yau moduli space*, *Nucl. Phys.* **B 338** (1990) 15.

[26] P. Candelas, X. De La Ossa, A. Font, S. Katz and D.R. Morrison, *Mirror symmetry for two parameter models, I*, *Nucl. Phys.* **B 416** (1994) 481 [hep-th/9308083].

[27] S. Hosono, A. Klemm, S. Theisen and S.T. Yau, *Mirror symmetry, mirror map and applications to Calabi-Yau hypersurfaces*, *Commun. Math. Phys.* **167** (1995) 301 [hep-th/9308122].

[28] S. Kachru and C. Vafa, *Exact results for $N = 2$ compactifications of heterotic strings*, *Nucl. Phys.* **B 450** (1995) 69 [hep-th/9505105].

[29] S. Kachru, A. Klemm, W. Lerche, P. Mayr and C. Vafa, *Nonperturbative results on the point particle limit of $N = 2$ heterotic string compactifications*, *Nucl. Phys.* **B 459** (1996) 537 [hep-th/9508155].

[30] A. Sen, *Orientifold limit of F-theory vacua*, *Phys. Rev.* **D 55** (1997) 7345 [hep-th/9702165].

[31] A. Klemm, B. Lian, S.S. Roan and S.T. Yau, *Calabi-Yau fourfolds for M- and F-theory compactifications*, *Nucl. Phys.* **B 518** (1998) 515 [hep-th/9701023].

[32] S. Sethi, C. Vafa and E. Witten, *Constraints on low-dimensional string compactifications*, *Nucl. Phys.* **B 480** (1996) 213 [hep-th/9606122].

[33] P. Berglund et al., *Periods for Calabi-Yau and Landau-Ginzburg vacua*, *Nucl. Phys.* **B 419** (1994) 352 [hep-th/9308005].

[34] P. Candelas, X. de la Ossa and F. Rodriguez-Villegas, *Calabi-Yau manifolds over finite fields, I*, hep-th/0012233.

[35] A. Strominger, *Special geometry*, *Commun. Math. Phys.* **133** (1990) 163;
P. Candelas and X. de la Ossa, *Moduli space of Calabi-Yau manifolds*, *Nucl. Phys.* **B 355** (1991) 455.

[36] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219.

[37] G. Dvali and S.H. Tye, *Brane inflation*, *Phys. Lett.* **B 450** (1999) 72, [hep-th/98124839812483];
S.H.S. Alexander, *Inflation from D - anti-D brane annihilation*, *Phys. Rev.* **D 65** (2002) 023507 [hep-th/0105032];
G.R. Dvali, Q. Shafi and S. Solganik, *D-brane inflation*, hep-th/0105203;
C.P. Burgess et al., *The inflationary brane-antibrane universe*, *J. High Energy Phys.* **07** (2001) 047 [hep-th/0105204];
G. Shiu and S.H.H. Tye, *Some aspects of brane inflation*, *Phys. Lett.* **B 516** (2001) 421 [hep-th/0106274].

[38] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055];
E. Silverstein and D. Tong, *Scalar speed limits and cosmology: acceleration from D-cceleration*, hep-th/0310221;
J.P. Hsu, R. Kallosh and S. Prokushkin, *On brane inflation with volume stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077];
H. Firouzjahi and S.H.H. Tye, *Closer towards inflation in string theory*, *Phys. Lett.* **B 584** (2004) 147 [hep-th/0312020].

[39] A. Maloney, E. Silverstein and A. Strominger, *de Sitter space in noncritical string theory*, hep-th/0205316.